DISCRETE MATHEMATICS

ASSIGNMENT-1 (Do Any 7)

Note: - First Question Compulsory

1. Explain: -

- Discrete Mathematics
- Sets
- Universal Set, Empty Set, Subset, Powerset, Cardinality of a Set
- Explain pigeonhole principle?
- permutation and combination with example

2. Explain all difference operations of Sets with example?
3. Explain all Propositional Logic? Show that $\sim p \land (\sim q \land r) \lor (q \land r) \lor (p \land r) \equiv r$
4. Show that proposition $\sim(p \land q)$ and $\sim p \lor \sim q$ are logically equivalent?
5. Explain principle of Inclusion and Exclusion with an example?
6. Explain Principle of MATHEMATICAL INDUCTION?
7. In how many ways can a team of 11 crickets be chosen from 6 bowlers, 4 wicket keepers and 11 batsmen to given a majority of batsmen if atleast 4 bowlers are to be included and there is one wicket keeper?

<u>ASSIGNEMENT-2</u>

(DO ALL QUESTIONS)

1. What do you understand by a function in discrete math? Explain the types of functions with examples.
2. What is a partially ordered set (POSET)? Provide an example.
3. Discuss the concept of a distributive lattice. How does it differ from a general lattice? Provide a formal definition and explain how the distributive property works in lattices with suitable examples.
4. Define Boolean Algebra and write all Properties of Boolean Algebra?

   ❖ Using Boolean identities, reduce the given Boolean expression:

   - $F(X, Y, Z) = X'Y + YZ' + YZ + XY'Z'$
   - $F(P, Q, R) = (P+Q)(P+R)$
   - $(A + B' + C')(A + B' + C)(A + B + C')$
   - $(AB'(C+BD) + A'B')C.$
   - $A = XY + X(Y+Z) + Y(Y+Z)$

5. What are recurrence relations? Discuss the methods used to solve linear, first-order recurrence relations with constant coefficients. Provide detailed steps and examples to illustrate the solution process.
6. What are generating functions in discrete mathematics? Explain their role in solving recurrence relations and provide examples of how they are used to find closed-form solutions for sequences.
7. Explain how recurrence relations are used to determine the efficiency of divide-and-conquer algorithms.

8. Using the Master Theorem, solve the recurrence relation $T(n)=3T(n/3) + n^2$. Describe the steps involved and discuss the implications of the result on algorithm performance.

9. Describe the Master Theorem and its applications in solving recurrences for divide-and-conquer algorithms.

10. Solve the recurrence relation $T(n)=T(n/2)+n\log n$ using the Master Theorem. Explain the method and interpret the solution in terms of algorithm efficiency.

ASSIGNMENT - 03

UNIT – III

VERY SHORT ANSWER QUESTIONS: (DO ALL QUESTIONS)

1. Define a semi-group.
2. What is a monoid?
3. What is the identity element in a group?
4. Define a subgroup.
5. What is a left coset?
6. Define a permutation group.
7. What is a homomorphism in group theory?
8. Define a normal subgroup.
9. State Lagrange's Theorem.
10. What is a quotient group?

SHORT ANSWER QUESTIONS: (DO ANY 8 QUESTIONS)

1. Explain the difference between a semi-group and a monoid.
2. What is the significance of the identity element in a group?
3. Prove that the identity element in a group is unique.
4. Define isomorphism between two groups. Provide an example.
5. Discuss the concept of inverse in a group.
6. Explain with an example how a coset is formed.
7. What is Cayley's Theorem?
8. Describe the relationship between normal subgroups and quotient groups.
9. Explain the role of group theory in coding theory.
10. What is the order of an element in a group?

LONG ANSWER QUESTIONS: (DO ANY 5 QUESTIONS)

1. Prove that every group has a unique identity element and each element has a unique inverse.
2. Explain the concept of isomorphism between two groups in detail. Provide an example with proof.
3. State and prove Lagrange's Theorem. Discuss its significance in the context of subgroup orders.
4. Define and explain permutation groups with an example.
5. What is a normal subgroup? How does it relate to the concept of a quotient group?
6. Explain Cayley's Theorem and its implications in group theory.
7. How does group theory apply to the design of coding schemes? Provide examples.
8. Define and differentiate between left and right cosets. Provide suitable examples.
9. Discuss the relationship between group homomorphisms and normal subgroups.
10. Explain the application of group theory in cryptography and coding theory.

ASSIGNMENT - 04

UNIT – IV

VERY SHORT QUESTIONS (DO ALL QUESTIONS)

1. Define a vertex.
2. What is an edge?
3. State Euler's formula.
4. What is a planar graph?
5. Define chromatic number.
6. What is a Hamiltonian path?
7. What does DFS stand for?
8. What is a connected component?
9. Name one algorithm for finding the MST.
10. What is the significance of the Five Color Theorem?

SHORT QUESTIONS (DO ANY 8 QUESTIONS)

1. Explain the difference between Eulerian and Hamiltonian paths.
2. Describe the conditions for a graph to be planar.
3. Outline the steps of Dijkstra's algorithm.
4. What is the significance of the degree of a vertex in a graph?
5. How do you calculate the chromatic number of a graph?
6. Describe Prim's algorithm for finding an MST.
7. Explain the concept of connected components in a graph.
8. What is the time complexity of Kruskal's algorithm?
9. Provide an example of a graph that is not planar.
10. Describe the properties of trees in relation to graph theory.

LONG QUESTIONS (DO ANY 5 QUESTIONS)

1. Prove Euler's formula for a connected planar graph.
2. Explain the steps to construct a spanning tree using Kruskal's algorithm.
3. Discuss the applications of shortest path algorithms in real-world scenarios.
4. Provide a detailed explanation of the Five Color Theorem and its proof.
5. Describe how to identify connected components using BFS.
6. Compare and contrast DFS and BFS in terms of their implementations and applications.
7. Provide a step-by-step explanation of Bellman-Ford's algorithm for shortest paths.
8. Explain the conditions for a graph to have an Eulerian path and circuit, with examples.
9. Analyze the complexity of finding a minimal spanning tree using different algorithms.
10. Discuss the significance of the chromatic number in graph theory and its applications.